Problem Set 11
Optical Waveguides and Fibers (OWF)

will be discussed in the tutorial on February 3, 2016

On January 29, 2016 a Lab tour will take place from 11:30 to 12:15. The meeting point is going to be the seminar room 3.41. If you are interested, please indicate your participation by filling your name in the Doodle poll http://doodle.com/poll/qfz8nmghygm4x5ym at least a week in advance of the scheduled Lab tour date.

Exercise 1: Dispersion compensation in optical fiber networks

In the early 90s, fiber links were installed between major cities in Germany. At that time, only standard single-mode fibers (SMF) with $C_\lambda = 16\,\text{ps}/(\text{nm km})$ at $\lambda_0 = 1.55\,\mu\text{m}$ were used. For high data rates and long transmission distances, this dispersion is too high, therefore it is necessary to install dispersion-compensating fibers (DCF) with negative dispersion after regular distances. In this way, one obtains a system of the type SMF-DCF-SMF-DCF... .

a) Assume that a DCF with $C_\lambda = -76\,\text{ps}/(\text{nm km})$ should be installed after 100 km of SMF. How long should this DCF be?

b) Consider a Gaussian and chirp-free pulse with a temporal width at half of the peak power (FWHM) of 50 ps at a center wavelength of 1550 nm. This pulse is launched into two different fibers: an SMF of 100 km length, and a DCF having the length calculated in part a). Calculate the pulse duration at the output of both fibers. How do the two pulses differ?

c) Accidentally, DCF with $C_\lambda = -80\,\text{ps}/(\text{nm km})$ and a length as calculated in part a) were installed. How can this be compensated after five sections of SMF-DCF?

Exercise 2: Coupling to a low-index contrast waveguide

In this problem, we consider the power coupling efficiency between an integrated waveguide and a free-space beam. The waveguide is oriented along the $z-$direction. Assume that the waveguide has a low refractive index contrast such that the guided and radiative eigenmodes can be described by scalar functions $\underline{\Psi}_m(x,y)$ and $\underline{\Psi}_{\rho,\mu}(x,y)$ that represent the dominant E_x-components of the linearly polarized modes. The eigenmode expansion hence reads:

$$\Phi\,(x,y,z) = \sum_m a_m \underline{\Psi}_m(x,y)\,\mathrm{e}^{-\mathrm{j}\,\beta_m z} + \sum_\mu \int_\rho a_\mu\,(\rho)\,\underline{\Psi}_{\rho,\mu}(x,y) e^{-\mathrm{j}\,\beta_\mu(\rho) z}\,\mathrm{d}\,\rho.$$

All eigenmodes obey the simplified orthogonality relations:

$$\frac{\beta_\nu}{2\omega\mu_0}\iint_{-\infty}^{\infty}\underline{\Psi}_\nu(x,y)\,\underline{\Psi}^\star_\mu(x,y)\,\mathrm{d}\,x\,\mathrm{d}\,y = \mathcal{P}_\mu\delta_{\nu\mu}$$
$$\frac{\beta_{\rho,\nu}}{2\omega\mu_0}\iint_{-\infty}^{\infty}\underline{\Psi}_{\rho,\nu}(x,y)\,\underline{\Psi}^\star_\mu(x,y)\,\mathrm{d}\,x\,\mathrm{d}\,y = 0,$$

where $\mathcal{P}_\mu$ is given by:

$$\mathcal{P}_\mu = \frac{\beta_\mu}{2\omega\mu_0}\iint_{-\infty}^{\infty}\left|\underline{\mathbf{\Psi}}_\mu(x,y)\right|^2\mathrm{d}\,x\,\mathrm{d}\,y.$$

To couple light into the waveguide, the facet (located at $z = 0$) is illuminated with a linearly polarized monochromatic free-space beam. The E_x-component of the excitation is given by $\Phi(x,y,0)$. Assume that the waveguide has a low index of refraction such that the reflection at the facet surface can be neglected.

a) Derive an expression for the fundamental mode amplitude a_0.

b) What is the power carried by the fundamental mode? Note that the time-averaged power flux of the total E_x-polarized field distribution $\Phi(x, y, z)$ is given by:

$$P(z) = \frac{1}{2} \iint_{-\infty}^{\infty} \mathrm{Re}\left[\Phi\left(x, y, z\right) \frac{1}{\mathrm{j}\,\omega \mu_0} \frac{\partial \Phi^{\star}\left(x, y, z\right)}{\partial z}\right] \mathrm{d}\,x\,\mathrm{d}\,y.$$

c) The fraction of power which is coupled from the incident field to the fundamental waveguide mode (mode index 0) is given by the power coupling efficiency η_0. Derive an expression for η_0 depending on the incident excitation field $\Phi(x, y, 0)$ and the fundamental mode field $\underline{\Psi}_0(x, y)$. Assume that the power of the incident excitation field can be approximated by:

$$P(0) = \frac{\beta_0}{2\omega\mu_0} \iint_{-\infty}^{\infty} \left|\Phi\left(x, y, 0\right)\right|^2 \mathrm{d}\,x\,\mathrm{d}\,y.$$

d) Explain qualitatively what the incident field distribution should look like in order to obtain maximum coupling efficiency to the fundamental mode.

Questions and Comments:

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